Today we will discuss why ‘understanding’ rather than just ‘learning’ a concept is important. Most questions can be solved using different methods. Sometimes, a particular method seems really easy and quick and we tend to ‘learn’ it without actually knowing why we are doing what we are doing. We need to understand the strengths and the weaknesses of the method before we use it. Let me elaborate with an example.

Question: What is the probability that you will get a sum of 8 when you throw three dice simultaneously?

When you throw three dice simultaneously, you can obtain a total sum ranging from 3 (1 on each die) to 18 (6 on each die). Let’s consider all the possible sums that we can obtain:

Sum of 3: This can happen in only 1 way. Each die shows 1 in this case

Sum of 4: This can happen in 3 ways. One die shows 2 and the other two show 1 each. Any of the three dice could show 2 so there are a total of 3 ways of getting a sum of 4.

Sum of 5: There are different ways of obtaining a sum of 5:

1, 1, 3 – Any of the three dice could show 3 so there are a total of 3 ways of obtaining 5 in this way.

1, 2, 2 – Any of the three dice could show 1 so there are a total of 3 ways of obtaining 5 in this way.

Total number of ways of obtaining a sum of 5 = 3 + 3 = 6

Sum of 6: There are different ways of obtaining a sum of 6:

1, 1, 4 – Any of the three dice could show 4 so there are a total of 3 ways of obtaining 6 in this way.

1, 2, 3 – These 3 numbers could be arranged among the 3 dice in 3! ways (using basic counting principle). There are a total of 3! = 6 ways of obtaining 6 in this way.

2, 2, 2 – This can happen in only one way. All dice show 2.

Total number of ways of obtaining a sum of 6 = 3 + 6 + 1 = 10

Similarly, we can find the number of ways in which all other sums can be obtained. Below I will list the number of ways in each case.

Sum of 3: 1 way	Sum of 18: 1 way
Sum of 4: 3 ways	Sum of 17: 3 ways
Sum of 5: 6 ways	Sum of 16: 6 ways
Sum of 6: 10 ways	Sum of 15: 10 ways
Sum of 7: 15 ways	Sum of 14: 15 ways
Sum of 8: 21 ways	Sum of 13: 21 ways
Sum of 9: 25 ways	Sum of 12: 25 ways
Sum of 10: 27 ways	Sum of 11: 27 ways

Notice the symmetry here. There is only 1 way of obtaining 3 and only one way of obtaining 18. Similarly, there are 3 ways in which you can obtain 4 and 3 ways in which you can obtain 17. Is it a co-incidence? No. The second half of the column can be obtained by replacing 1 by 6, 2 by 5, 3 by 4, 4 by 3, 5 by 2 and 6 by 1. The number of ways of obtaining a sum is symmetrical about the center.

We have simplified one aspect of this problem. If we need to find the number of ways of obtaining 15, we can instead find the number of ways of obtaining a sum of 6 (which is psychologically easier to handle). Now the problem is whether there is an easier way of obtaining the number of ways in which you can get a sum of 6 or do we have to enumerate it in every case.

You might have come across something like this:

Sum of 3: 2C2 = 1 way

Sum of 4: 3C2 = 3 ways

Sum of 5: 4C2 = 6 ways

Sum of 6: 5C2 = 10 ways

Sum of 7: 6C2 = 15 ways

Sum of 8: 7C2 = 21 ways

Perfect till now! Matches the numbers we obtained above. It seems like a good method to use instead of writing down all the cases, doesn’t it? But what happens further on?

Sum of 9: 8C2 = 28 ways

Sum of 10: 9C2 = 36 ways

These don’t match! The key to understanding this is to understand why it works in the above given cases. First review this post.

Focus on method II of question 2. Notice how you divide n identical objects among m distinct groups. Let's take the example of a sum of 7. You have to divide 7 among 3 dice such that each die must have at least 1 (no die face can show 0). First step is to take 3 out of the 7 and give one each to the three dice. Now you have 4 left to distribute among 3 distinct groups such that it is possible that some groups may get none of the four. Think of partitioning 4 in 3 groups. This can be done in (4+2)!/4!*2! = 6C2 ways (check out the given link if you do not understand this)
This is how you obtain 6C2 for the sum of 7.
The concept works perfectly till the sum of 8. Thereafter it fails. Think why. I will explain it next week.